MODERN WAYS OF APPROACHING MATHEMATICS IN SCHOOLS

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Abstract: *This paper is intended to be a theoretical study that aims to to emphasize the positive aspects of the impressive and rapid knowledge that children and adolescents can access by clicking scientific sites which are loaded with proficient and highly quality validated information.*

Seeing things from another perspective, the information exposed here can be considered a study directly regarding the build up of the student's ability to use maieutic technology in a selfinduce manner and the discovering, step by step, the next concept or application related.

In this article we started from the halving rule and its applicability and validity in any field, and we have reached ***the irrational numbers***, ***φ*** *and* ***the pure knowledge of Pythagoras***, ***the π number***, ***pentagons and pentagrams***, ***the golden rectangle***, ***the order of Fibonacci****. It is only a step from here to the artistic domain, towards which the teenager is invited to acces on his own , through enjoyable and direct communication with the internet world.*

Keywords: *knowledge, science,usage of virtual network data, associative chain, creativity.*

We live in the fantastic era of the information adventure, in which specialists claim that in each 10-year time period the quantity of information doubles in each area of research. By applying the doubling law ($1+2+2^2+2^3+\ldots+2^n=2^{n+1}-1<2^{n+1}$), it results that in the last 10 years, our civilization has produced more 'science' than in all our past of scientific research from the stone age until today. This way, the human civilization owes more and more to science which each passing year.

The teaching process is, as such, subject to continuous change. By trying to adapt the educational programs and methods, we try to induce to the young learning skills as efficiently as possible. Generally, the school is created to sustain children at a medium level of knowledge and

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demands intellectual and personal skills that focus on the outside of the student, rather than on the inner side. Unnourished aptitudes and skills go to waste if not discovered in time. There is an explanation for why the number of high school students which choose performance decreases with age. Hereditary predispositions are open to the modeling influences of the socio-cultural activities. One man becomes accomplished through evaluating and processing what the educational system and the social environment has to offer. Some students are likely to read more, to study books of expertise on a certain domain, to do research on the internet, showing as such a high interest on a particular field of knowledge.

True culture one can only gain for oneself. School, even the superior one, only gives us a general direction in solving a problem. For our own practical needs, as well as for our own satisfactions we need deep knowledge that can only be achieved though individual work and study. A book, an advice given at the right time can be more valuable than a didactic activity planned on several years.[1]

We need to start from the perception and understanding level of the student and try to encourage the learning process as discretely as possible. The correct way of teaching is not to transmit to the students our own prejudices towards certain areas of study, because this could easily influence the natural path of his thinking process. On the contrary, it is better to keep track of what appeals to the child, of what he/she enjoys studying, to figure out and support what the child decides to embraces as a career. Showing devotion to the child will only lead to positive effects in the future. Advice can only come from wisdom and it implies ethics and a high moral value. It is hoped that each new generation will be better than the previous one. But who doesn't want that, as a parent and as a teacher?

The right conception about life is created, both as a mentality and as an ideal. Imagine a scale of knowledge on which the human, part earth part spirit keeps ascending. He is meant to always have this scale of values in front of him, which opens the way for greatness in knowledge.

Take for example Rudolph Steiner, whose wise modesty made him wait until he fully mastered the miracles of the modern science until he decided to turn to a more spiritual path.

Games have a very important role in developing the children's abilities.[2] Among the numerous qualities a game has, I will further mention: 1) it has an

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[1] Marcus, S., *Educația în spectacol*, Ed. Spandugino, Bucharest, 2010.

[2] Marcus, S., *Jocul ca libertate*, Ed. Scripta, Bucharest, 2003.

agreeable character, 2) its unpredictable nature, 3) the problematic aspect, 4) the strategic aspect. The first aspect is described by psychologists, and the second, third and forth aspects make subject for scientific disciplines such as: the information theory, artificial intelligence and the mathematical game theory. Through games, certain types of thinking are developed: logical thinking, combinatorial thinking, inductive, analogical, algorithmic, linguistic etc. Being by excellence a seeking activity, the game takes the mind from the known to the unknown, from predictable to unpredictable. Thus, we associate the game with a process of creation and sensibility. Furthermore, the French mathematician Evariste Galois once said: 'In vain do mathematicians want to hide. They do not prove, but combine and compare and only by wondering from one side to the other do they reach the truth.' A thorough understanding of a certain chapter, no matter how well written it is in a course book, can only be achieved by comparing multiple sources (course books, exercise books etc.)

The lecture must be made critically, despite how famous the author is. The understanding gained by defeating obstacles can only bring joy and it allows us to make connections between pieces of information separated until then. The historical method, as would the Romanian mathematician Dimitrie Pompeiu would say, is so useful and sometimes so fertile. To look back in time and figure out what it was and how things progressed is an undying spring of revival and reinforcement of our knowledge.

One good method to gain the students' attention towards a certain domain is to insert during classes several biographical facts of intellectual personalities and of the brilliant ways that they have figured in order to give a solution to the problems they were faced with. This is one way in which the school can encourage and control the use of information from various websites.

At class level, individual activity portfolios can be created, encouraging this way team work by grouping the students passionate about the same domain together. Creativeness is developed this way as well as the relationships between students and between the students and the teacher. There are many unsolved problems in sciences and every day new ones seem to appear. It is our duty as teachers and the duty of the future generations to solve them.

I will exemplify the way of approaching different knowledge on the internet through some mathematics lessons. We will stop at the algebraic notion of an irrational number and we will continue the discussion with the famous 'golden number', φ.

It is known that, until Pythagoras, the mathematicians were using the knowledge regarding mathematics in almost all of their daily issues. Proclus said that Pythagoras was the first mathematician to raise mathematics to the level of an accessible field. Mathematics was, for Pythagoras and his apprentices, pure knowledge. Moreover, later, the scientific results discovered by them found their applicability in various activities. Regarding the mystical aspect, the numbers stood for:

1 - The origin of everything in the Universe, 2 – female, 3 – male, 4- the harmony between the four elements (earth, air, fire, water), 10 = 1+2+3+4 which is a triangular number. They knew ten stars (Sun, Moon, Venus, Mercury, Venus, Earth, Mars, Jupiter, Saturn, the Inner Fire ≠ Sun, and Anti-Earth, always hidden behind the Sun).

Iamblichus (242 – 327 a.d.), who wrote nine books about the Pythagoras sect, describes the way in which he discovered the fundamental principles of the musical harmony. By touching a lira chord, it produces a standard keynote or a tone on the full length of the vibrating chord. The harmonious tones are only in particular spots: 1/2, 1/4, 1/5. In any other spots, if we fix the node, it generates a tone which is in disharmony with the rest. Pythagoras discovered, for the first time, a mathematical law governing a physical phenomenon and proved that there is a fundamental law between mathematics and science. After this discovery, the science people have always searched the mathematical laws related to physical processes, in any natural phenomena.

For example, the well-known π number (the ratio between the circle's length and its diameter) is, from now on, used in another field. The mineralogist Hans Henik Stolum, from the Cambridge University, calculated the ratio between the real length of a river and its straight-line length, unveiling that is approximately equal to π.

The name of incommensurable dimensions is known in Greek as alogon (something that cannot be explained through logos or speech as the ratio between two integers).

Hippasus of Metapontion (Magna Greece, Metaponto, Italy today) belonged to the school of Pythagoras. He is granted with the discovery of the irrational numbers. He was severely punished (expulsed from the sect or even killed by drowning) for divulging secrets of a discovery made by Pythagoras' sect. (The construction of a regular dodecahedron inscribed in a sphere and the golden ratio in the pentagon's construction).

In a regular pentagon's ABCDE, by drawing all the diagonals we get a starred pentagon (pentacle) which also results in a new regular pentagon

$A_1B_1C_1D_1E_1$. After n steps, the regular pentagons $A_iB_iC_iD_iE_i$, $1 \leq i \leq n$, determines a pentacle and the procedure is repeated to the infinite. From the equal ratio string: $AC/DE=A_1C_1/D_1E_1=\ldots=A_nC_n/D_nE_n=\ldots$ turns out that the ratio between the length of a diagonal and the length of an edge is an irrational number, φ.

Euclid, in his famous book 'The Elements' proves the existence of exactly five regular polyhedrons: the tetrahedron, the cube, the octahedron, the dodecahedron and the icosahedrons. The last two imply the regular pentagon. Regular pentagons are directly related to what Euclid called 'dividing into medium and extreme ratio'. Let there be a segment of length 1. Determine on this segment a point C as to AB/AC=AC/BC, which is equivalent with the first segment being in the same ratio with the bigger segment, as the bigger segment is to the smaller one.

Let AC=x, it results that $\varphi=1/x=x/1-x$ or $x^2=1-x$ from which $x= (-1+\sqrt{5})/2$ from which we can now find the value for $\varphi = 1/x= (1+\sqrt{5})/2=1, 6180339887\ldots$ It is the positive solution to the equation $x^2-x-1=0$.

According to the string of ratios above, it results that φ is an irrational number (the numbers after the, never ending). It is the first irrational number found and defined throughout history.

We can easily see that $\{\varphi\}= \varphi-1 = 1/\varphi$, meaning that φ and 1/φ have the same irrational part.

Let ABCD be a rectangle in which AB/AC=φ, meaning it is a 'golden rectangle'. Inside it, we will build a square with the length of one side equal to AC. Thus, a new 'golden rectangle' is determined EBDF and inside EBDF, the process can continue unlimitedly. By building circle sectors with the centers in F, G and so on the logarithmical spiral is generated, which can also be found in he structures of the Universe or in the human body, in painting, sculpture, architecture etc. We are therefore both the masters and the slaves of the golden number. Architects and artists have designed their masterpieces accordingly to the golden ratio, believing that it gives the work a more pleasant aspect. (Φ is the first letter in the famous' Greek sculptor Phidias (490-430 B.C.), famous for using the 'golden number' in his works: Zeus of Olympia, Athena Parthenos from Athens. During the renaissance, great artists like Leonardo da Vinci, Pierro della Francesca, Albert Dűrer have made use of the 'golden number'. The german philosopher Fechner G.T. (1801-1887), the father of psychophysics, has tested in the 19th century a few hundred persons. The

test consisted in choosing the favorite rectangle from a series of rectangles displayed. The most favorite was the one whose ratio between length and width was closest to φ.

An interesting connection can be found between the number φ and Fibonacci's string. Leonard of Pisa (1170-1250), also known as Leonard Fibonacci (Filius Bonacci, his father, a very kind person, had the gift of cutting rectangular surfaces in mosaics with minimum material lost) introduced to Europe the Arabic set of numbers in his work 'Liber Abacci' (1202, 1228). In his second work, the one from 1228, can be found the rabbit-multiplying problem. "How many pairs of mature rabbits are born after 12 months knowing that at the start we had one pair, the gestation period is one month and each pair reaches maturity after one month?"

It can be easily verified that the first 12 terms of the sequence are:

1,1,2,3,5,8,13,21,34,55,89,144. It is proved that it represents the organic growth.

Let the Fibonacci string be noted as $(f_n)_{n\geq 1}$. It is noticed that: $f_1=f_2=1$, $f_{n+2}=f_{n+1}+f_n$, for any $n\geq 1$.

This is a recurrent string linear and homogenous of second degree. The general term represents the Binet formula:

$f_n=1/\sqrt{5}(\varphi^n-\psi^n)$ where ψ is the quadratic conjugate of φ , $\psi=1/2(1-\sqrt{5})$

There are many proprieties of the Fibonacci string and therefore we open a new, attractive and important research subject for the students. For example the limit of the string f_{n+1}/f_n is φ. It can be proven that the substrings of odd or even degree of the proportion string: $a_n=f_{n+1}/f_n$ are exactly increasing or decreasing, both of them being convergent to φ.

Another application of the golden number is in the numbers theory. Thereby the number φ can be expressed through a continuous infinite fraction that uses only the digit.

As it is mentioned in work,[3] there are two numbers of the "Revue d'Esthétique" (1961) magazine dedicated to the link between art and science. Most of the articles are dedicated to the golden number. Notable contribution in this area was given by Matila Ghika, 30 years ago. Engineer, graduate in law and literature, M. Ghica elaborated a pioneering creation in the domain of aesthetics trying to make deep correspondences between scientific thinking and the artistic one. He developed a long lasting activity in diplomacy, to become an aesthetics teacher, in the U.S.A during The Second World War, and then in London.

[3] Marcus, S., *Artă şi Ştiinţă*, Ed. Eminescu, 1986.

The old desideratum of Leibniz, wishing to build a universal language, was a topic of great interest for the Romanian Pius Servien (Şerban Coculescu (1903-1959), the son the the astronomer Nicolae Coculescu). He finished a doctorate degree for literature in Paris and he had broad of knowledge in the science field. He anticipated main ideas of today mathematics' linguistics. Let's stop here and elaborate some conclusions.

Positive Aspects

- It is to be notice that through mentioning one subject (the golden ratio) to our students, we were invited to read and discover so many applications and to make connections between completely distinct notions. Therefore, the internet is a continuously invitation to acquaintance.
- We find new sourced of information
- We can notice an approach between the scientific and the artistic way of thinking.
- The internet is used by the students in a positive way.
- The growth of the lesson's quality in the organized school environment.

(In the year 1995, a student of IXth grade, who became a doctor, observed that $f_1=1^2$, $f_{12}=12^2$ and asked if the diofantic equation $f_n=n^2$ has other distinct solutions, than n=1, n=12?

He received the answer that the equation has a finite number of solutions because $f_n/n^2\to\infty$, therefore, for example in the vicinity $V=(2,\infty)$ there will be all the terms of the string, excepting a finite number.

The search for all the solutions leads to a little study demonstration that these are the unique solutions.) Hereby, the exchange of ideas between students and teachers or between students, is encouraged. While experimenting on groups of students it was observed their interest for studing mathematics and especially their applications in diverse areas of study.

- An open attitude for subjects related to mathematics or for other domains of knowledge.
- It is a useful exercise of reading and writing in languages of international interest.
- It is noticed the high level necessary for the admission to international universities. Take as an example the subject given in the last 3 years at the admission exams at the universities for France.

- Using courses from the international universities leads to acquiring a universal scientific language.
- Maintaining relationships with the students learning abroad, determine a better feed-back for both the Romanian teachers and for the students who wish to pursue an international education.

Negative aspects

- The isolation from the family, caused by an exaggerate amount of time spent on using the computer.
- Reading on a screen can cause tiredness.
- The laconic and schematic way of expression the teenagers use on the chat leads to an atrophy of the language.

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